Abstract

A well trained cyclist may apply up to two times his weight on a structure of no more than eight/nine hundreds grams. It is evident that the frame of the bicycle is becoming more and more strategic and since the eighties the most important manufacturers are working with Finite Element techniques to design their frames: at the beginning using “beam” elements to optimize the cross sectional shapes of the main tubes and, later on, using “shell” elements to analyze fillets and tube junctions. Very recently, also optimization tools are becoming available which, in principle, appear very promising to get an optimum frame design. In this paper the application of a software based on a gradient method to the optimization of a bicycle frame is described. The target of the job was to improve the performance, in terms of lateral stiffness, of an existing frame playing with tubes shapes and keeping the same weight of the structure. The frame has been described using a parametric approach and then the typical loads which correspond to the action of pedalling have been applied. First, the structure has been optimized to get the same behaviour when pushing on the left pedal and on the right one (considering the chain traction) and then the final optimization process has been performed.

1. Introduction

Bicycle frame, even if it is a structure not so complex, has requirements for low weight/stiffness ratio which make the design more challenging than it may appear at first view and for this reason the use of the Finite Element Method has been a common tool in the design of high performance bicycle frames since ’80.

P.S Soden and M.A: Millar [1] used “beam” elements to evaluate stresses of a steel bicycle frame during common cycling situations and compared the FEM results with the experimental ones based on strain-gauges measurements.


In parallel to this, a deeper understandings of load applied to a bicycle frame during pedalling has become available. Among the most significant papers on this subject, Soden and Adeyefa [3] computed forces and moments applied by a 80 Kg cyclist in three typical circumstances (starting, speeding, hill climbing ) and C. Stone and M. Hull took measurements on a treadmill using a special bike equipped with strain-gauges.

Similar information can be found in many text book and manuals,[5],[6],[7],[8],[9],[10],[11],[12],[13],[14].

Those activities proposed FEM as an instrument to improve frame performances in term of strength, high stiffness, low weight by means of a better knowledge of stresses.

The designer, using FEM, knows which part of the frame is more stressed and could modify the thickness of the tube or its mean diameter.

This approach, thought correct, can be now considered restrictive essentially for two reasons:
1. The structure of the frames is hyper static so, modifying a tube, the distribution of stresses over the whole structure is changed, so it difficult to predict the correct change to make;
2. In this way the number of iteration FEM analysis-design modification-FEM analysis is limited and the designer can evaluate only few modifications. That is restrictive especially for modern carbon fibre frames whose technology allows a great freedom in term of tube curvature and cross sectional shape.

More recently, also optimization tools are becoming more user-friendly and appear very promising to get an optimum frame design.

The finite element method coupled to optimization algorithm allows the designer to evaluate a great number of frame configuration.

The designer can include in the optimization process a great number of design variables associated to the thickness of tubes, to their shapes and to the material property also in term of orientation of the carbon fibers to obtain an objective under certain structural constraints.

The objective can be, for example, weight reduction of the frame maintaining the same stiffness or the increase of stiffness at the same weight.

The optimization task of this activity was the reduction of the difference of lateral deformation of the frame during pedalling. In fact the lateral deflection of a conventional bicycle frame characterized by a structure with one plane of symmetry is higher when the rider pushes on the right pedal.

In this paper the various phases of the activity will be described.

First the realization of the analysis model i.e. the fem model including boundary condition will be presented.

Then the result of the static analysis will be showed.

Then the set-up of the design model will be presented including the realization of shape-modifying design variables, the introduction of the objective function and constraints.

Finally it will show the results of the optimization in term of improvement of the design objective and shape modification.

2. Analysis model

The building-up of the analysis model has been based on the representation of real bicycle frame. The middle surface, given by the constructor, have been used to create a shell mesh of the frame while the others component of the bicycle i.e. cranks, handlebar, seat tube and front fork have been represented by “beam” elements.(Fig.1).
Fig. 1 The finite element model to optimize
The load condition used in the analysis are shown in Fig. 2.

Fig. 2 The set of forces considered
The constraints applied to the model are:

1. The central node of the front fork has two d.o.f. constrained: $U_z = U_y = 0$ allowing only displacements in the direction of motion;
2. The central node of posterior wheel fork have been constrained to have $U_x = U_y = U_z = \Phi_x = 0$

The properties of element in term of elastic behaviour, thickness, cross section has been a simplified representation of the real ones following the indications given by the constructor.

The great part of the activity have been focused on the modifying of the frame shape in order to reduce the differences in lateral deflection during pedalling.

Observing more carefully the load conditions associated to pedalling you can see that the condition relative to push on right pedal end push on left pedal are asymmetric.

To evaluate the different behaviour of the frame to pedalling loads a static analysis on the model was run.
The results in term of structure deformation are shown in figure 3 (X25) while the lateral displacement of the bottom bracket for the four different cases of pedalling together with the absolute value of their difference are listened in table 1. In the entire paper results will be shown in normalised form referred to the maximum deflection in the baseline.

![Deformation of the frame(X25) for the sub case relative to push on left pedal (left) and to right pedal (right).](image)

<table>
<thead>
<tr>
<th>Standing pedalling</th>
<th>Sitting pedalling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uz r (baseline)</td>
<td>Uz l (normalised)</td>
</tr>
<tr>
<td>0,35</td>
<td>0,06</td>
</tr>
</tbody>
</table>

Table 1 The lateral displacement of bottom bracket for pedalling and the difference between push on right pedal and push on left pedal. Values have been normalised with respect to the baseline.

2. Shape Optimization: model parameterization

Some general concept about optimization and then to the basic concepts of shape optimization inside HyperWorks will be presented.

A general optimization problem can be described by the following equations:

\[
\min (f(\alpha)) = \min (f(\alpha_1, \alpha_2, \ldots, \alpha_n))
\]

Subject to:

\[
g_j(\alpha) \leq 0 \quad j = 1, \ldots, m
\]

\[
\alpha_i^L \leq \alpha_i \leq \alpha_i^U \quad i = 1, \ldots, n
\]
The objective function $f(\alpha)$ and the constraint functions $g(\alpha)$ are structural responses obtained from a finite element analysis. The dependency of both objective function and constrain functions from the design variable vector $\bar{\alpha}$ is unknown and so the solution can be found only using approximated methods.

The optimization algorithm of OptiStruct is a gradient-based algorithm finding a local minimum of the objective function in the design space defined by the constraints equations, through the iterative application of the following steps:

1. Analysis of the physical problem using finite elements;
2. Convergence test, whether or not the convergence is achieved;
3. Design sensitivity analysis;
4. Solution of an approximate optimization problem formulated using the sensitivity information;
5. Back to 1;

This approach is based on the assumption that only small changes occur in the design with each optimization step. The result is a local minimum. The biggest changes occur in the first few optimization steps and, as a result, not many system analyses are necessary in practical applications.

The design sensitivity analysis of the structural responses (with respect to the design variables) is one of the most important ingredients to take the step from a simple design variation to a computational optimization.

The design update is computed using the solution of an approximate optimization problem, which is established using the sensitivity information. In fact OptiStruct using the information of the sensitivity analysis decide the direction of movement inside the design space i.e. the new set of design variable values and then new configuration is valued through a new static analysis (step1).

If convergence is achieved the procedure stops, if not, there is a new sensitivity analysis.

The selection of the vector of design variables $\bar{\alpha}$ depends on the type of optimization being performed. In topology optimization, the design variables are element densities. In size optimization (including free-size), the design variables are properties of structural elements. In topography and shape (including free-shape) optimization, the design variables are the factors in a linear combination of shape perturbations.

OptiStruct has the capability of performing shape optimization. In shape optimization, the outer boundary of the structure is modified to solve the optimization problem.

Shape variables are defined in OptiStruct in a way very similar to that of other shape optimization codes. Each shape variable is defined by using a DESVAR bulk data entry. DVGRID bulk data entries define how much a particular grid point location is changed by the design variable. Any number of DVGRID bulk data entries can be added to the model. Each DVGRID bulk data entry must reference an existing DESVAR bulk data entry if it is to be a part of the optimization. The DVGRID data in OptiStruct contains grid location perturbations, not basis shapes.

The generation of the design variables and of the DVGRID bulk data entries is facilitated by the HyperMorph utility, which is part of the Altair HyperMesh software.

The shape of a general finite element model is completely defined by the vector of nodal coordinates $\bar{X}$. 
The shape optimization in OptiStruct is based on the Perturbation Vector approach.

Using HyperMorph and choosing an HyperMorph strategy (domains and handle, morphvolumes, freehand) the user can alter the shape of the mesh resulting in a final form \( \overline{X} \):

\[
\overline{X} = \overline{X}_0 + \overline{PV}
\]

In which \( \overline{PV} \) is the perturbation vector created with HyperMorph and contains the three component of the displacement of each node associated to the user-defined mesh alteration:

\[
\overline{PV} = \begin{bmatrix}
\Delta x_1 \\
\Delta y_1 \\
\vdots \\
\Delta x_n
\end{bmatrix}
\]

The component of the perturbation vector are the coefficient of the DVGRID data entry.

Setting-up the design model the user associate the perturbation vectors created to a design variable and the final form will be a linear combination of the initial form \( \overline{X}_0 \) and of the perturbation vectors with coefficients the values of the design variables found by OptiStruct:

\[
\overline{X} = \overline{X}_0 + \sum_{i=1}^{j} \alpha_i \cdot \overline{PV}_i
\]

In this activity the shape optimization aim is to modify the cross section and curvature of each component of the frame in order to obtain a reduction of the differences in lateral deformation during pedalling.

To give a great freedom to the solver in creating the new shape 100 perturbations vector which may be associated to shape optimization design variables have been created. Fig. 4 shows some of the design variables relatives to the chain stays and give an idea of the perturbation vector and so design variables created for the whole frame.
Figure 4 Some of the shape variables of the model

Associating a perturbation vector to a design variable it is important to chose the correct range of variation of each design variable i.e. the value of $\alpha^l_i$ and $\alpha^u_i$ respectively the lower and the upper bound.

In fact giving too large range of variation can determine a final shape excessively strange and technologically unfeasible.

At the same time in our work the new shape must not create interference with the other component of the bicycle and in particular the transmission system, the cranks, the seat tube and the wheels.

In order to have a visual reference, auxiliary surfaces relatives to those components were created in the model.
The upper and lower bound of each design variable have been chosen to avoid interference with those surfaces even if it is impossible to guess how they will combine in the final shape. At the same time there are constraints on the shape imposed by U.C.I. (Union Ciclyste Internationale) This are illustrated in appendix A. In order to obtain the minimization of the difference in lateral displacement a second level response was created:

\[ f_{obb} = \left| U_{z_{pR}} \right| - \left| U_{z_{pL}} \right| \]

Where \( U_{z_{pR}} \) is the lateral displacement relative to standing, push right pedal and \( U_{z_{pL}} \) is the lateral displacement relative to sub case standing, push left pedal. Even if the difference in lateral displacement appears also in the sitting pedalling sub cases we have decided to build the objective function on the standing pedalling sub cases because they recorded greater difference in lateral displacement. The lateral displacement relative to standing sub cases will be still monitored to evaluate the improvement during optimization. The optimization constraint functions change in the several optimization attempt except the mass constraint, that must be always under or equal to initial mass of 990g.
3 The results of Optimization

In this section the results relative to the various attempt made to optimize this structure will be presented. Critically analysing the results of each attempt new constraint equations was introduced or the lateral bounds of variables modified.

In the first optimization the following objective and constraints have been used:

**Objective:** \( \text{min}(f_{obb}) \)
**Constraints:** \( \text{mass} \leq 990 \text{g (initial value)} \)

The results shows an improvement on the objective of 40,3%, but it is principally due to the rise of displacement relative to left push, so even if the frame is more balanced it is also less stiff. So a new constraint function was introduced in order to avoid the frame to became excessively compliant. It has been decided to constraint the sum of compliance i.e. elastic deformation energy of the sub cases relative to standing pedalling.

So, in the second optimization the following objective and constraints have been used:

**Objective:** \( \text{min}(f_{obb}) \)
**Constraints:** \( \text{mass} \leq 990 \text{g (initial value)} \)
\( C_{ppR}+C_{ppL} \leq 8 \text{J (initial value)} \)

The whole results are listen in the following table:

<table>
<thead>
<tr>
<th>Standing pedalling</th>
<th>Sitting pedalling</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_z^r ) (baseline)</td>
<td>( U_z^l ) (normalised)</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>It: 0</td>
<td>-1</td>
</tr>
<tr>
<td>It: 12</td>
<td>-0,87</td>
</tr>
<tr>
<td>( \Delta % )</td>
<td>-13%</td>
</tr>
</tbody>
</table>

Table 2 the result of second optimization, Values have been normalised with respect to the baseline.

Even if the reduction in the objective function is inferior to the case above (39,1% against 40,3%) the objective is obtained symmetrically because the displacement relative to left push rises from 0,35(normalised value) to 0,41 (+0,11) while the displacement relative to right push reduces from 1(baseline) to 0.87 (-0,13).

At this point it has been decided to introduce in the optimization process an improvement in comfort by reducing vertical stiffness of the frame measured through the vertical displacement of the seat node in the sub case road irregularity. The reduction of vertical stiffness is introduced as constraint through two explorative attempt: reduction \( \geq 10\% \) (case1), reduction \( \geq 20\% \) (case2) . So, in the third optimization the following objective and constraints have been used:

**Objective:** \( \text{min}(f_{obb}) \)
**Constraints:** \( \text{mass} \leq 990 \text{g (initial value)} \)
\( C_{ppR}+C_{ppL} \leq 8 \text{J (initial value)} \)
Vertical stiffness reduction \( \geq 10\% \) (case1)
Vertical stiffness reduction \( \geq 20\% \) (case2)

The results are listen in the following table:
Table 3 Results of case 1. Values have been normalised with respect to the baseline.

<table>
<thead>
<tr>
<th></th>
<th>Standing pedalling</th>
<th>Sitting pedalling</th>
<th>Vert Stiff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uz r (baseline)</td>
<td>Uz l (normalised)</td>
<td>F_obj (normalised)</td>
</tr>
<tr>
<td>It: 0</td>
<td>-1</td>
<td>0,35</td>
<td>0,65</td>
</tr>
<tr>
<td>It: 9</td>
<td>-0,86</td>
<td>0,47</td>
<td>0,39</td>
</tr>
<tr>
<td>δ %</td>
<td>+14%</td>
<td>+35%</td>
<td>-40,3%</td>
</tr>
</tbody>
</table>

Table 4 Results of case 2. Values have been normalised with respect to the baseline.

<table>
<thead>
<tr>
<th></th>
<th>Standing pedalling</th>
<th>Sitting pedalling</th>
<th>Vert Stiff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uz r (baseline)</td>
<td>Uz l (normalised)</td>
<td>F_obj (normalised)</td>
</tr>
<tr>
<td>It: 0</td>
<td>-1</td>
<td>0,35</td>
<td>0,65</td>
</tr>
<tr>
<td>It: 11</td>
<td>-0,88</td>
<td>0,45</td>
<td>0,43</td>
</tr>
<tr>
<td>δ %</td>
<td>-12%</td>
<td>+28%</td>
<td>-33%</td>
</tr>
</tbody>
</table>

In the case 1 the reduction in vertical stiffness is -11% and, at the same time, there is a reduction of the objective function of 40,3%.
In the case 2 the reduction in vertical stiffness is -20% and while there is a reduction of the objective function of 33%.
The main task of this activity is to reduce the difference in lateral displacement of the frame so we have decided to evaluate better the new shape proposed by case 1.
The final shape (iteration 9) was loaded in the HM model to evaluate if there was interferences with other components and if the shape violates the UCI rule.
The final shape had two problems:

1. the right part of chain stays shows interference with cranks and transmission system (Fig.6);
2. The left part of seat stays do not include a straight line as imposed by UCI (Fig.7);

Figure 6 Interference with crank (bigger surface) and transmission
Figure 7 Violation of UCI rules

It has been decided to review the upper limit of the design variables relative to these components and to rise the limit of other variables to give new freedom to optimization.
So, in the **fourth optimization**, the range of variables has been modified while the objective and constraints has been the same of third optimization-case1:

**Objective:** \[ \text{min}(f_{\text{obb}}) \]  
**Constraints:** \[ \text{mass}<990\text{g}(\text{initial value}) \]  
\[ C_{ppR}+C_{ppL} < 8J(\text{initial value}) \]  
\[ \text{Vertical stiffness reduction}>10\% \]

The results are listen in the following table:

<table>
<thead>
<tr>
<th>It: 0</th>
<th>It: 9</th>
<th>∆%</th>
<th>Standing pedalling</th>
<th>Sitting pedalling</th>
<th>Vert Stiff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Uz r (baseline)</td>
<td>Uz sx (normalised)</td>
<td>F_obj (normalised)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1</td>
<td>0.35</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+12%</td>
<td>-0.87</td>
<td>0.47</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+36%</td>
<td>+12%</td>
<td>+36%</td>
<td>-39%</td>
</tr>
</tbody>
</table>

Table 5 Results of the fourth optimization. Values have been normalised with respect to the baseline.

Observing table 5, you can see that the objective function reduces less that case1(-39%) but the new shape have no problem of interference and violation of UCI rules (Fig.8,9) At the same time the reduction of difference in lateral displacement is obtained in a symmetrical way in fact the displacement relative to left push rises from 0.35 to 0.47 (+0.12) while the displacement relative to right push reduces from 1 to 0.87 (-0.13).

The new shape (right) compared to the old(left) are shown in figure 10.

The main differences introduced by optimization are:

1. the cross section relative to chain stays rises especially in the right part;
2. the cross section in the seat stays changes continuously and the two tube display a curvature out of vertical plane;

The geometry of the new shape have been created internally to HyperMesh applying the optimization results to the FE model and the using the tool: create surfaces from FE
The output is a .iges file which will be passed to the designer.

![Figure 10 Comparison between initial frame shape(left) and optimized shape (right).](image)

4. Conclusions
The computer aided optimization of a bicycle frame has been described. Analysing various papers about forces on the bicycle six representative frame load conditions have been identified and the activity has been concentrated on pedalling. The unsymmetrical load of the chain causes a difference on lateral deformation of the frame during pedalling.
In fact the lateral deflection of a conventional bicycle frame characterized by a structure with one plane of symmetry is higher when the rider pushes on the right pedal. A preliminary static analysis on the FEM model representing a real carbon fiber bicycle frame confirmed that the lateral deformation of the frame when the rider pushes on right pedal (standing) is higher (lateral displacement of the bottom bracket, normalised value: right push=1, left push =0,35).
Then the model was parameterized in order to create shape design variables to obtain by the computer aided optimization a new frame more balanced in term of lateral deformation during pedalling. The final shape obtained by optimization reduces the difference in lateral deformation during pedalling (-39%). At the same time the new frame is more comfortable by reducing vertical stiffness(-11%).
The new shape has no problem of interference with other component of the bicycle and comply with the UCI rules. Although a great attention was spent in creating PV to have an agreeable new shape, the final geometry created by the software have to be reviewed by the designer in order to obtain a more “attractive” frame. At the same time a future development could be a “size” optimization of the frame including also thickness between design variables.
The results show the efficacy of the computer aided optimization tool in the improvement of the frame, especially if you consider that at the same time it is possible to evaluate many possible shape modification and to evaluate in a short time if the new form has problem of interferences or violation of UCI rules.
5. References


Appendix A: UCI requirement for bicycles: “Configuration”

UCI Cycling regulation, *Part 1 General organization of cycling as a sport*, Pages 61-62

1.3.020 c) Configuration

For road competitions other than time trials and for cyclo-cross competitions, the frame of the bicycle shall be of a traditional pattern, i.e. built around a main triangle. It shall be constructed of straight or tapered tubular elements (which may be round, oval, flattened, teardrop shaped or otherwise in cross-section) such that the form of each element encloses a straight line. The elements of the frame shall be laid out such that the joining points shall follow the following pattern: the top tube (1) connects the top of the head tube (2) to the top of the seat tube (4); the seat tube (from which the seat post shall extend) shall connect to the bottom bracket shell; the down tube (3) shall connect the bottom bracket shell to the bottom of the head tube. The rear triangles shall be formed by the chain stays (6), the seat stays (5) and the seat tube (4) with the seat stays anchored to the seat tube at points falling within the limits laid down for the slope of the top tube.

The maximum height of the elements shall be 8 cm and the minimum width 2.5 cm. The minimum width shall be reduced to 1 cm for the chain stays (6) and the seat stays (5). The minimum thickness of the elements of the front fork shall be 1 cm; these may be straight or curved (7). (See diagram «Shape (1)»). The top tube may slope, provided that this element fits within a horizontal template defined by a maximum height of 16 cm and a minimum thickness of 2.5 cm.

(text modified on 7.06.00; 1.01.05).
1.3.021 For road time trials and for track competitions, the elements of the bicycle frame may be tubular or solid, assembled or cast in a single piece in any form (including arches, cradles, beams or any other). These elements, including the bottom bracket shell, shall fit within a template of the "triangular