Finite element simulation of impact problems

Some comparison results between FER/Impact and RADIOSS

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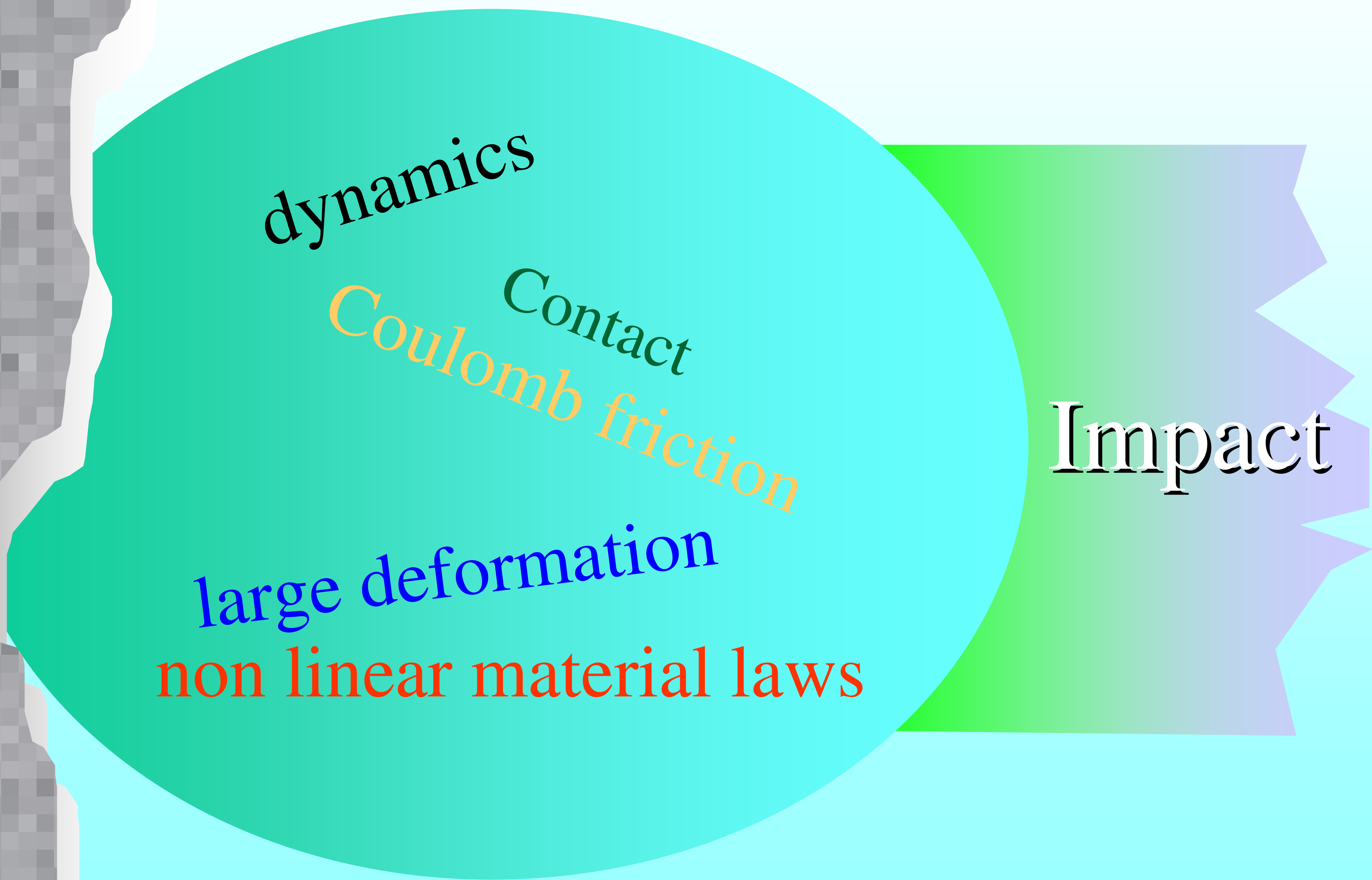
Outline

- Introduction
- Solution algorithms
- Numerical examples
- Conclusions and ongoing work

Introduction: Modeling of impact problems



Introduction: Difficulty

Local level

- Integration of nonlinear constitutive laws
- Integration of contact and friction laws

Global level

- Solution of the equilibrium equation together with the contact inequality
- Integration of the equation of motion taking into account impact phenomena

Introduction: Coupling

- Solution methods of nonlinear equations
  - Newton-Raphson
  - BFGS, Riks
- Time stepping algorithms
  - Explicit scheme
  - Implicit scheme: Newmark, Houbolt, HHT, ...
  - Scheme of first order or second order
- Solution methods of contact problems
  - Penalty, Lagrange multipliers, flexibility,
  - Augmented Lagrangian method, bi-potential method, ...

Introduction: **Objectives**

- Develop an efficient algorithm for frictional contact/impact problems: Bi-First algorithm
- Develop a software to deal with general problems:
  - *contact between two or more deformable or rigid bodies, in static or dynamic case*
  - *contact with small or large strain in 2D or 3D*
  - *contact with isotropic or orthotropic friction, …*
- Apply the developed method to industrial problems (metal forming, joints, impact, tire, brake, …)

Solution Algorithm
"Bi-First"

- *ISM - Implicit Standard Material*
  Contact Bi-potential
- *ALM - Augmented Lagrangian Method*
- *Uzawa/Newton Algorithm*
- *Flexibility method*
- *First order time stepping*
- *OOP in C++*

ISM - Implicit Standard Materials (De Saxcé & Feng, 1991)

- Generalized standard materials (associated rule)
  - Legendre-Fenchel inequality: $V(\mathbf{x}) + W(\mathbf{y}) \geq \mathbf{x} \cdot \mathbf{y}$
  - material law: $\mathbf{y} \in \partial V(\mathbf{x})$ explicit
- Non standard materials (soils, dry friction, composite, ...)
  - Bi-potential inequality: $b(\mathbf{x},\mathbf{y}) \geq \mathbf{x} \cdot \mathbf{y}$
  - material law: $\mathbf{y} \in \partial_x b(\mathbf{x},\mathbf{y})$ implicit

Contact bi-potential

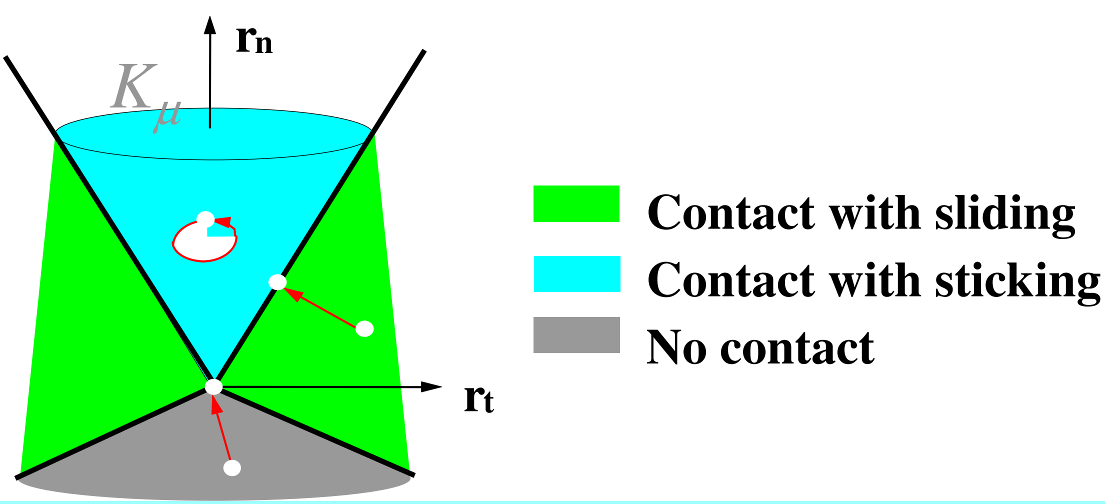
$$b_c(-\dot{\mathbf{u}},\mathbf{r}) = \bigcup_{\Re_-}(-\dot{u}_n) + \bigcup_{K_\mu}(\mathbf{r}) + \mu r_n \left|-\dot{\mathbf{u}}_t\right|$$

Local algorithm

- **ALM : r =** ***proj*** **[** $(r_n+\rho_n(x_n-\mu|\mathbf{v}_t|), \mathbf{r}_t+\rho_t\mathbf{u}_t), K_\mu$ **]**
- **Uzawa :**

- predictor $\mathbf{r}^{*i+1} = \mathbf{r}^i - \rho^i\left(\mathbf{v}_t^i + (x_n^i - \mu|\mathbf{v}_t^i|)\mathbf{n}\right)$

- corrector $\mathbf{r}^{i+1} = \textbf{\textit{proj}}\,(\mathbf{r}^{*i+1}, K_\mu)$



$$\mathrm{Proj}_{\mathbf{K}_\mu}(\mathbf{r}^*) = \mathbf{r}^* \quad \text{if } \|\mathbf{r}_t^*\| < \mu r_n^*$$

$$\mathrm{Proj}_{\mathbf{K}_\mu}(\mathbf{r}^*) = \mathbf{0} \quad \text{if } \mu\|\mathbf{r}_t^*\| < -r_n^*$$

$$\mathrm{Proj}_{\mathbf{K}_\mu}(\mathbf{r}^*) = \mathbf{r}^* - \left(\frac{\|\mathbf{r}_t^*\| - \mu r_n^*}{1+\mu^2}\right)\left(\frac{\mathbf{r}_t^*}{\|\mathbf{r}_t^*\|} - \mu\mathbf{n}\right)$$

Newton algorithm : Joli & Feng (IJNME 2008)

Implicit time integration (FER/Impact)

Non-linear dynamic behavior of solid with contact is governed by

$$\mathbf{M}\,\ddot{\mathbf{u}} = \mathbf{F} + \mathbf{R}_c \qquad \mathbf{F} = \mathbf{F}_{ext} + \mathbf{F}_{int} - \mathbf{C}\dot{\mathbf{u}}$$

first order scheme

$$\mathbf{u}^{t+\Delta t} - \mathbf{u}^{t} = \Delta t\left((1-\theta)\,\dot{\mathbf{u}}^{t} + \theta\,\dot{\mathbf{u}}^{t+\Delta t}\right)$$

Jean (1989); Wronski (1994), ...

Difficulties

$\Delta\mathbf{u}$ and $\mathbf{R}_c$ are both unknown

Multiple non-linearities:

- material (constitutive laws)
- geometrical (large displacement)
- contact and friction (inequality)

N-R iterative process:

$$\begin{cases} \hat{\mathbf{K}}^{i}\Delta\mathbf{u} = \hat{\mathbf{F}}^{i} + \mathbf{R}_{c}^{\,i+1} \\ \mathbf{u}^{i+1} = \mathbf{u}^{i} + \Delta\mathbf{u} \end{cases}$$

Key idea: separation of non-linearities

- Compute locally the contact reaction forces
- Compute globally the displacements and velocities

Remark

This method neither changes the global stiffness matrix, nor increases the degrees of freedom, as opposed to the penalty method or Lagrange multiplier method.

Explicit time integration (RADIOSS)

Second-order central difference scheme

$$\dot{\mathbf{U}}_t = \frac{1}{2\Delta t}(\mathbf{U}_{t+\Delta t} - \mathbf{U}_{t-\Delta t})$$

$$\ddot{\mathbf{U}}_t = \frac{1}{\Delta t^2}(\mathbf{U}_{t+\Delta t} - 2\mathbf{U}_t + \mathbf{U}_{t-\Delta t})$$

$$\mathbf{M}\,\ddot{\mathbf{u}} = \mathbf{F} + \mathbf{R}_c$$

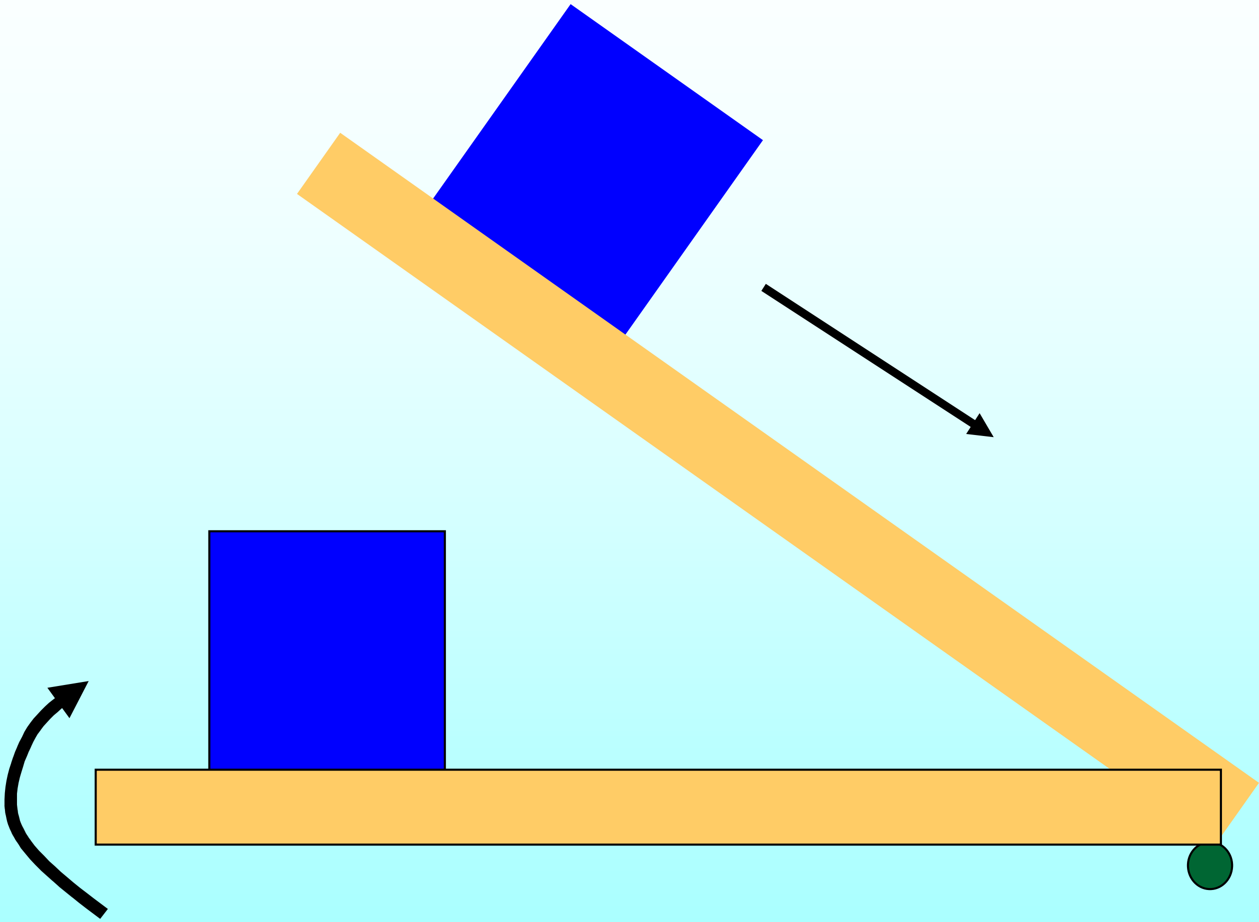
$$\mathbf{U}_{t+\Delta t} = \Delta t^2 \mathbf{M}^{-1}((\mathbf{F}_{ext})_t - (\mathbf{F}_{int})_t + \mathbf{R}_{t+\Delta t}) + 2\mathbf{U}_t - \mathbf{U}_{t-\Delta t}$$

The mass matrix **M** is diagonal (obtained by mass lumping), the solution is so obtained without solving a system of equations and without checking the convergence.

The computational cost per time step is much less than for an implicit method.

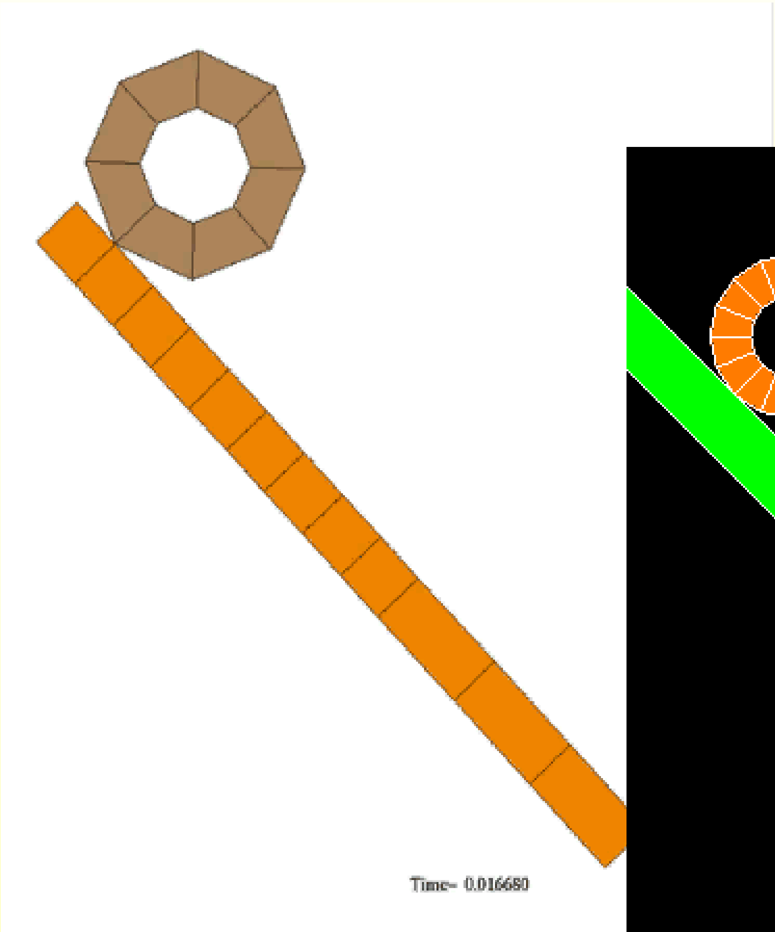
Some examples

Coulomb test

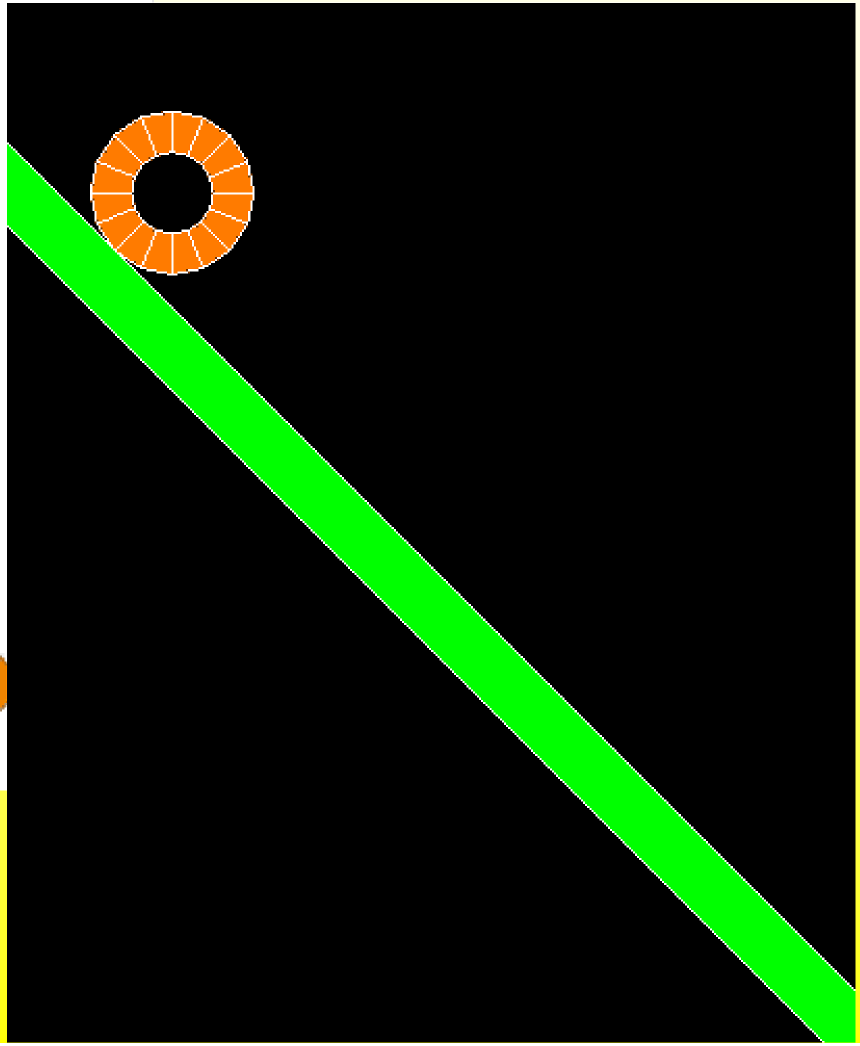


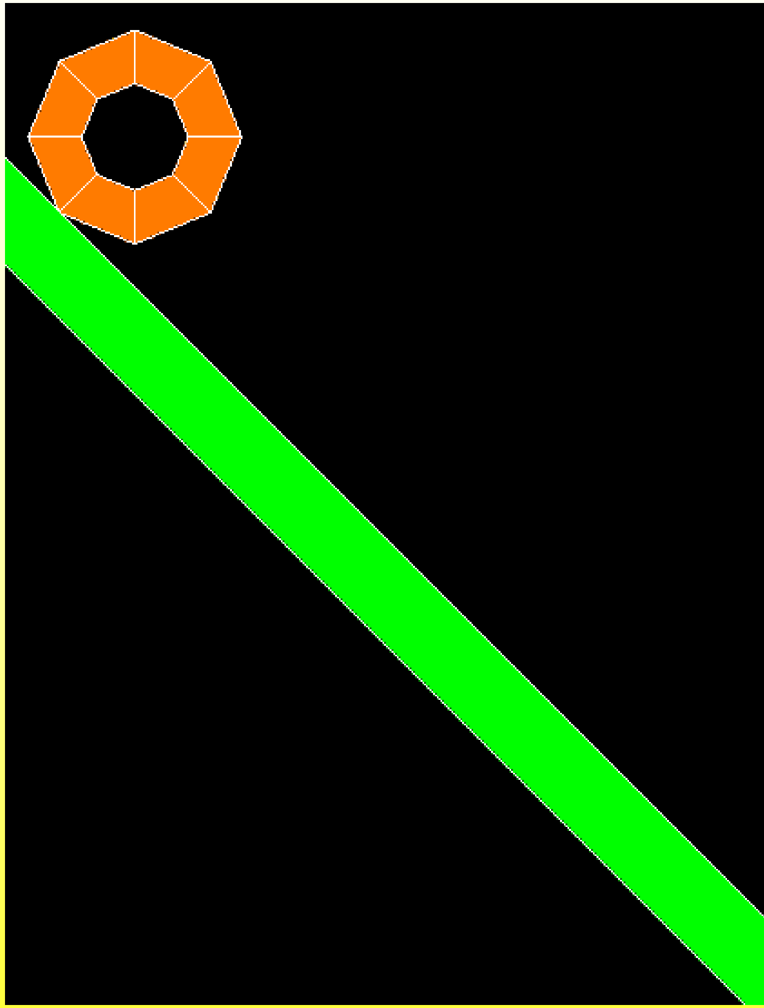
Demo with
FER/View

Rolling contact



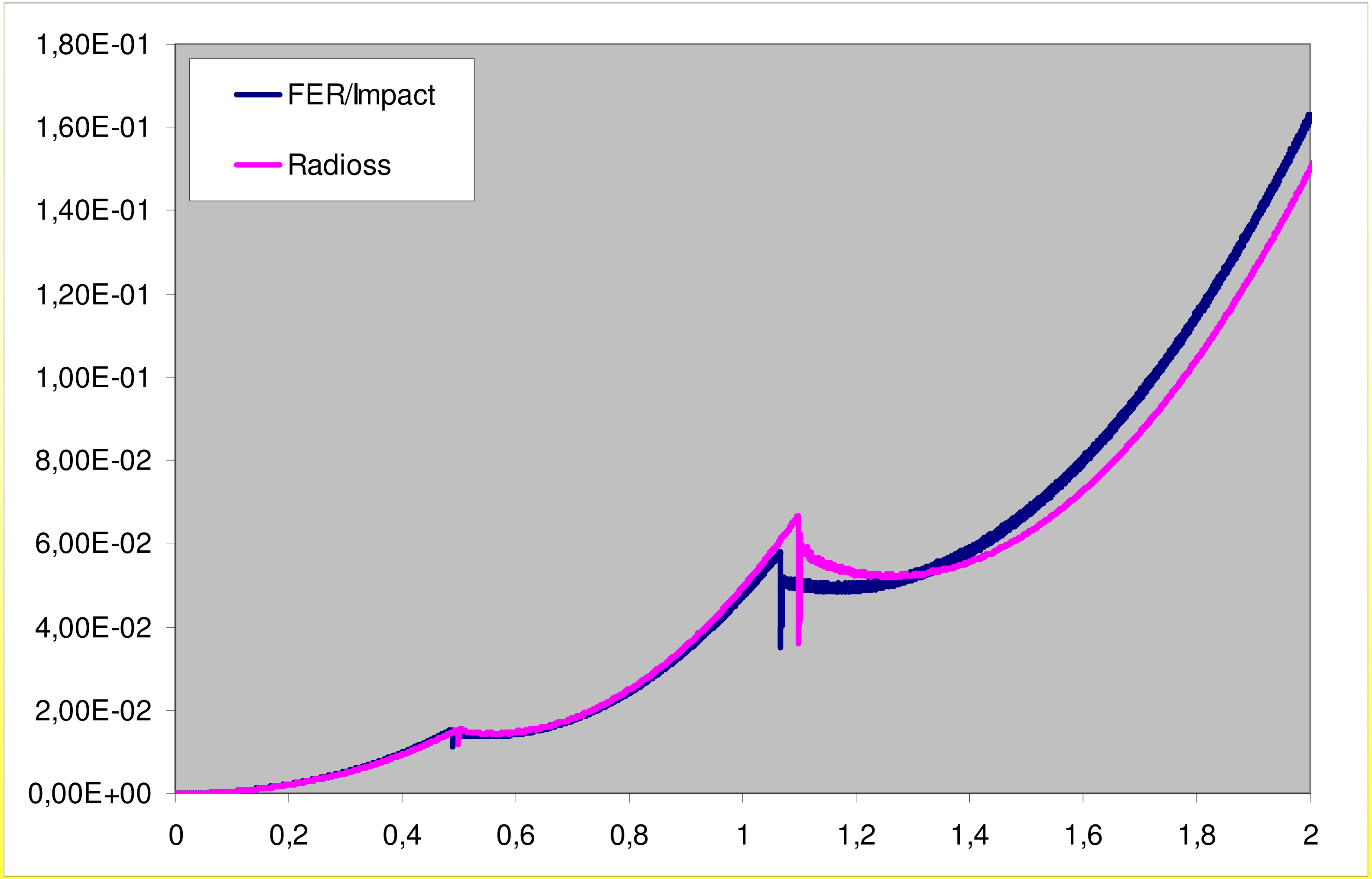
Radioss



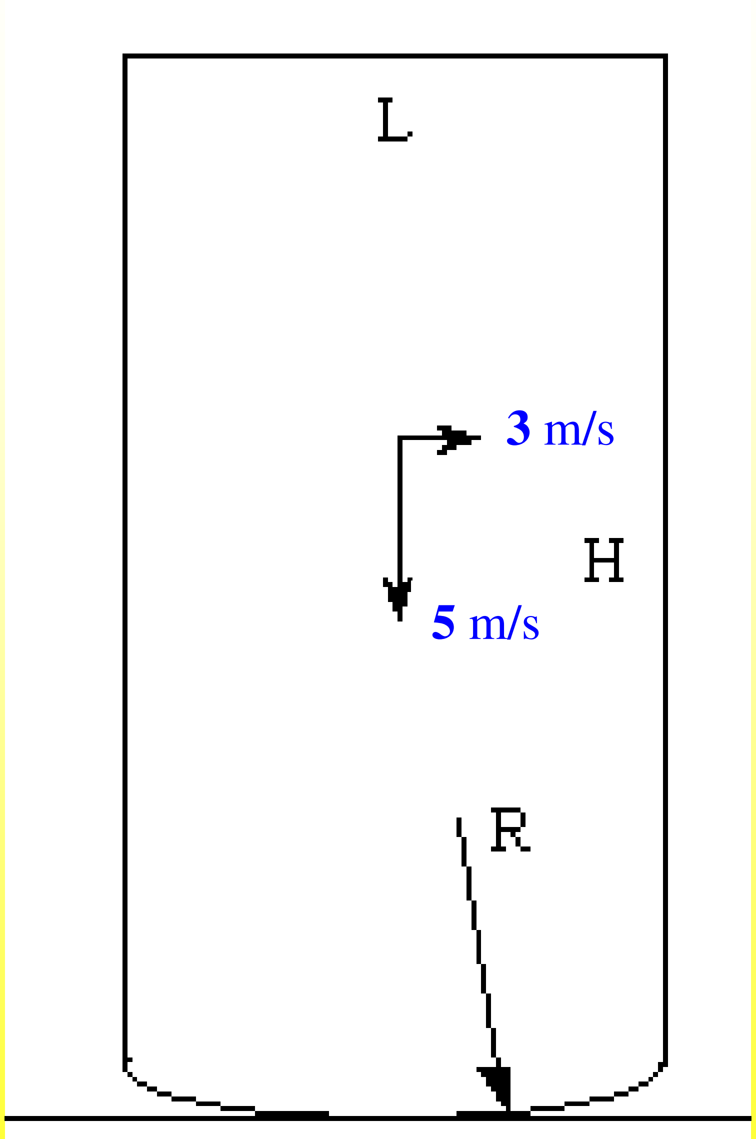


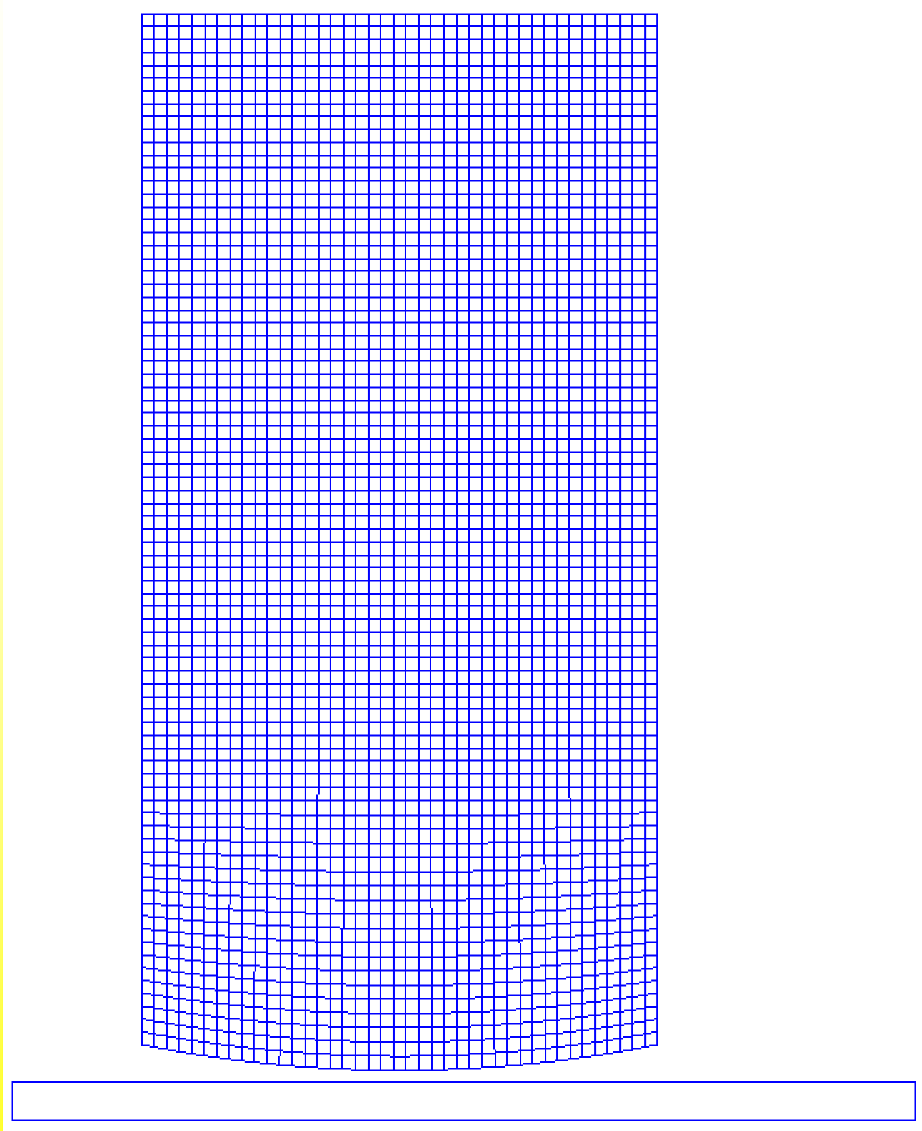
FER/Impact

Kinetic energy



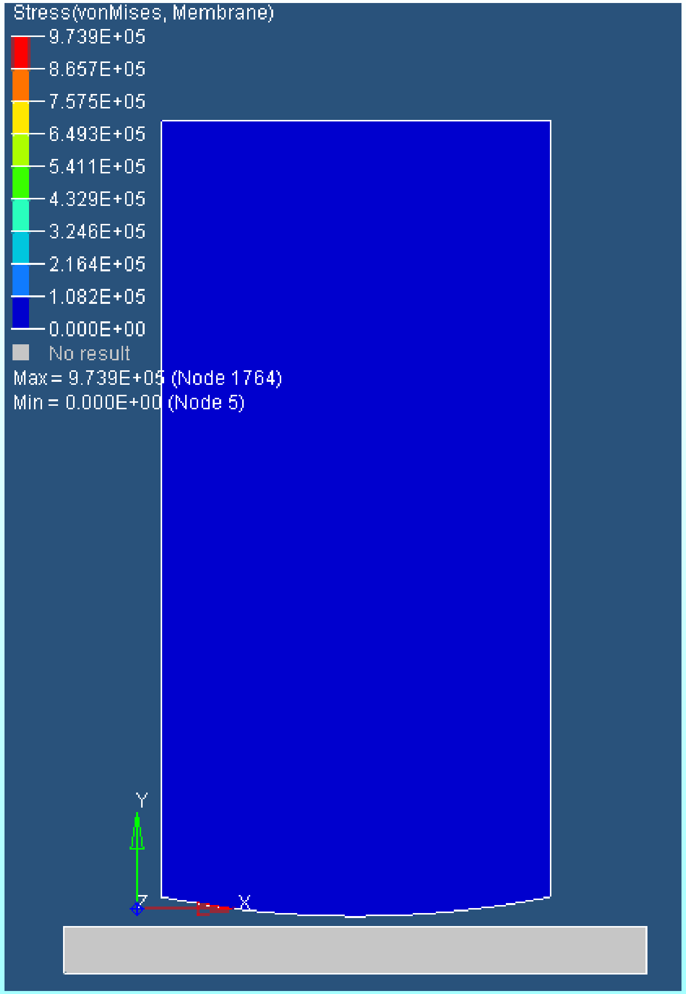
Oblique impact of an elastic plate

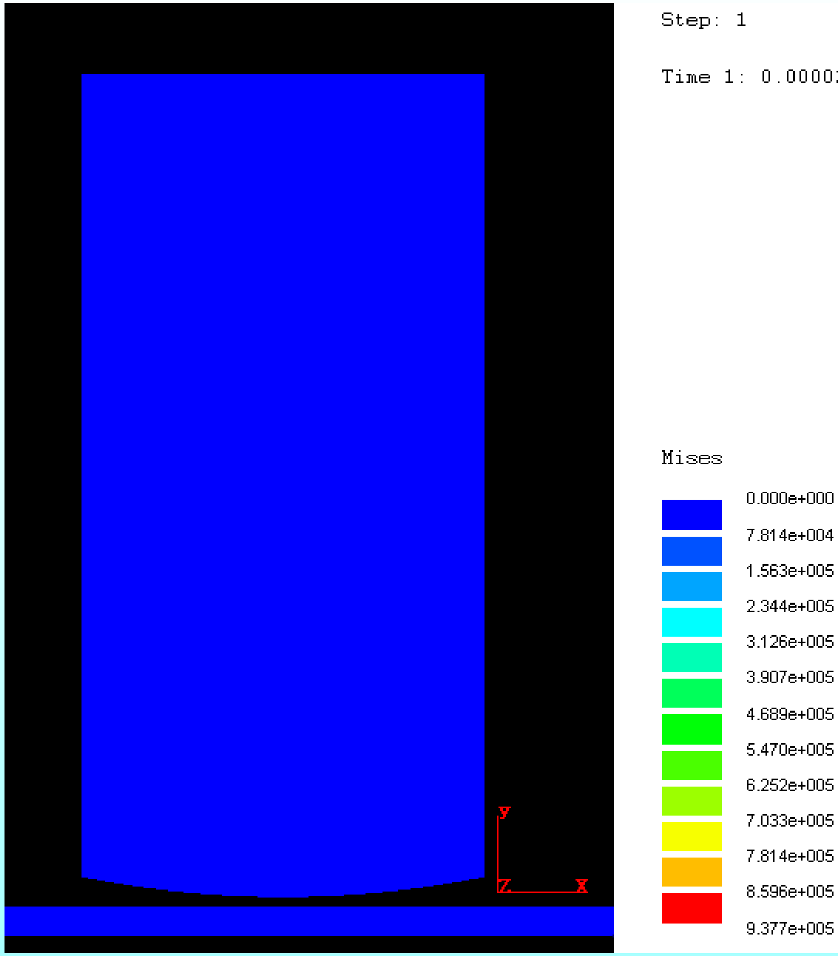




41 × 80 elements

Comparison





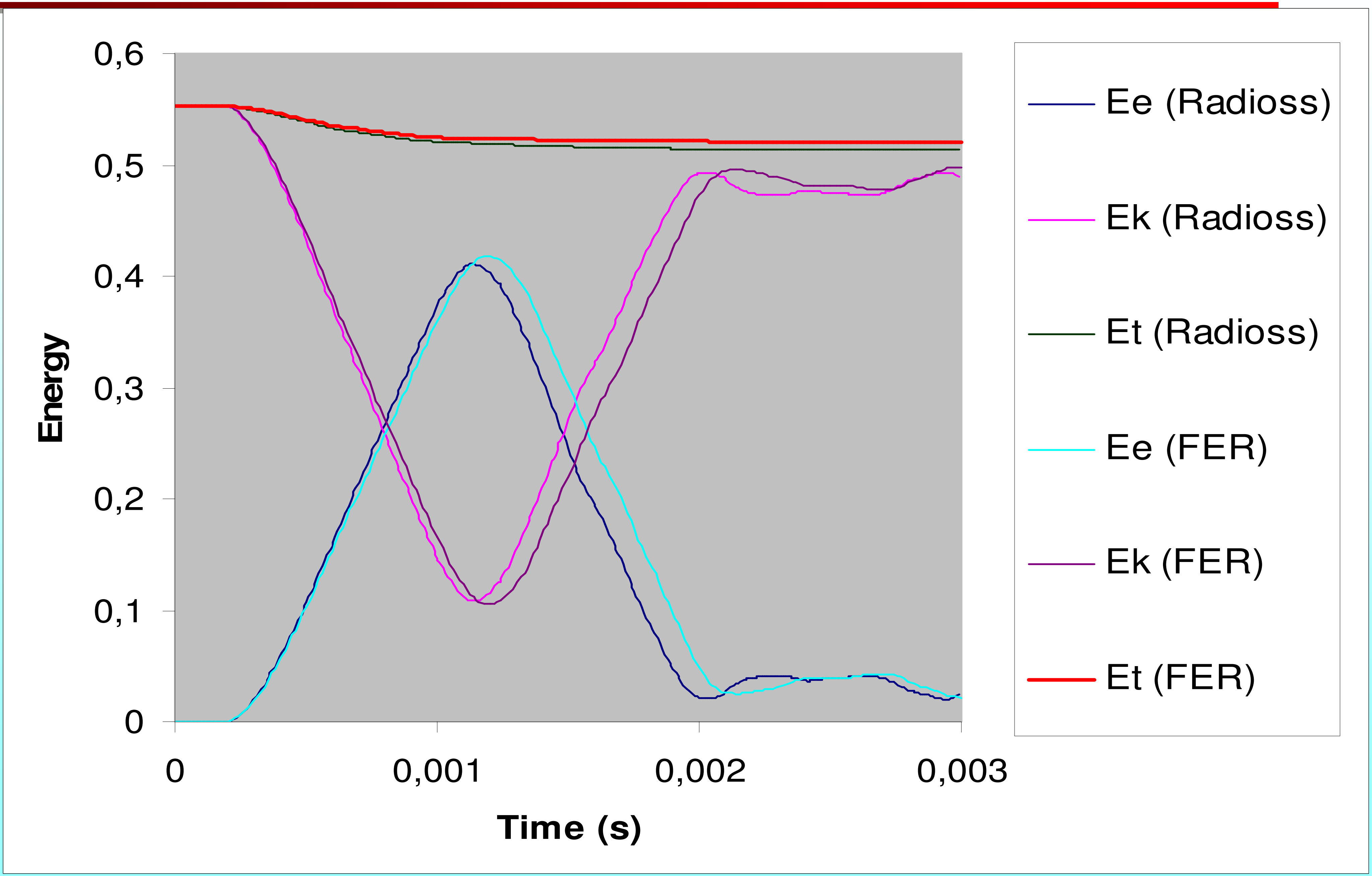
Radioss

$\Delta t \approx 10^{-7}$ depending of the gap

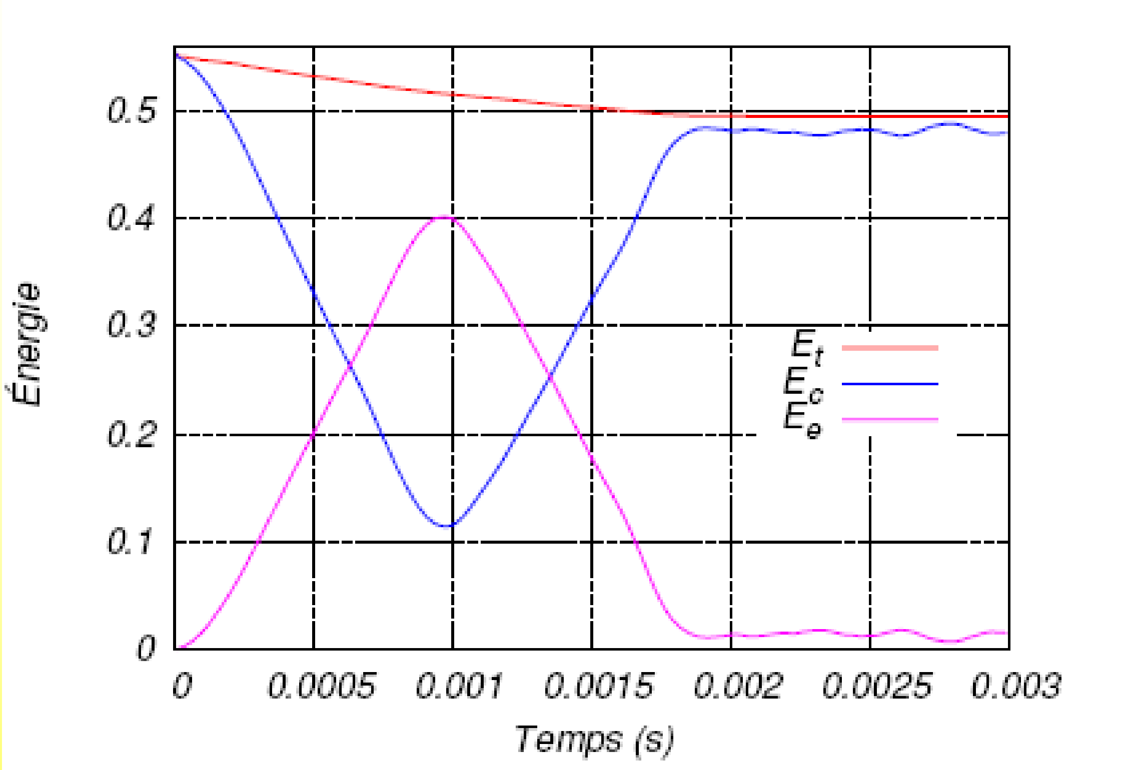
FER/Impact

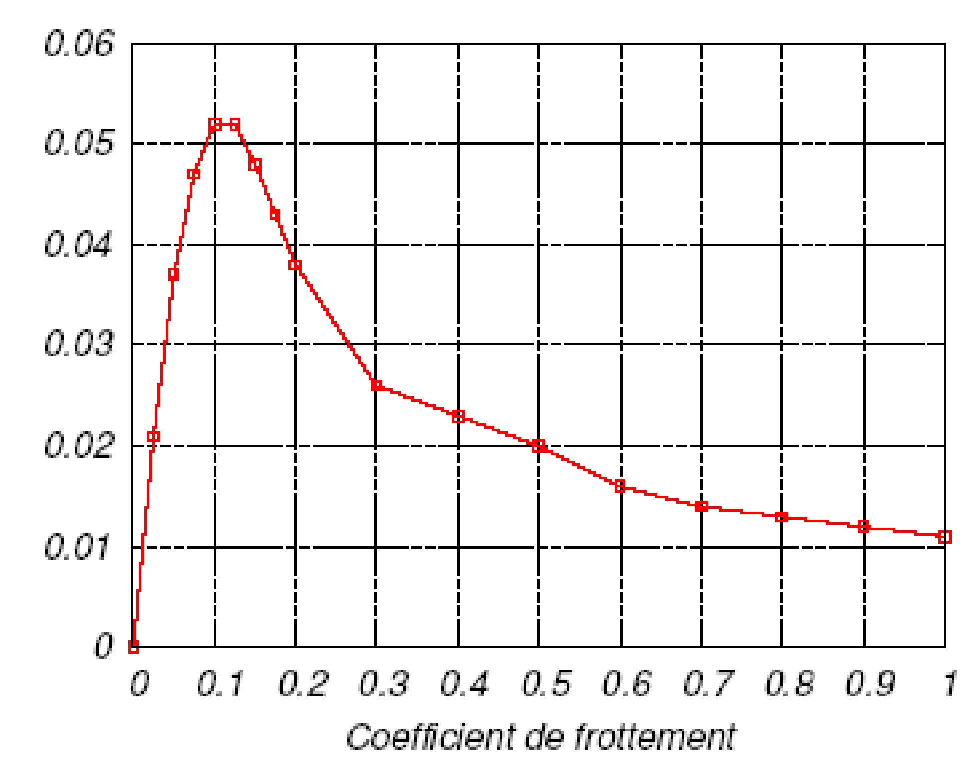
$\Delta t = 10^{-5}$

Energy evolution



Energy evolution

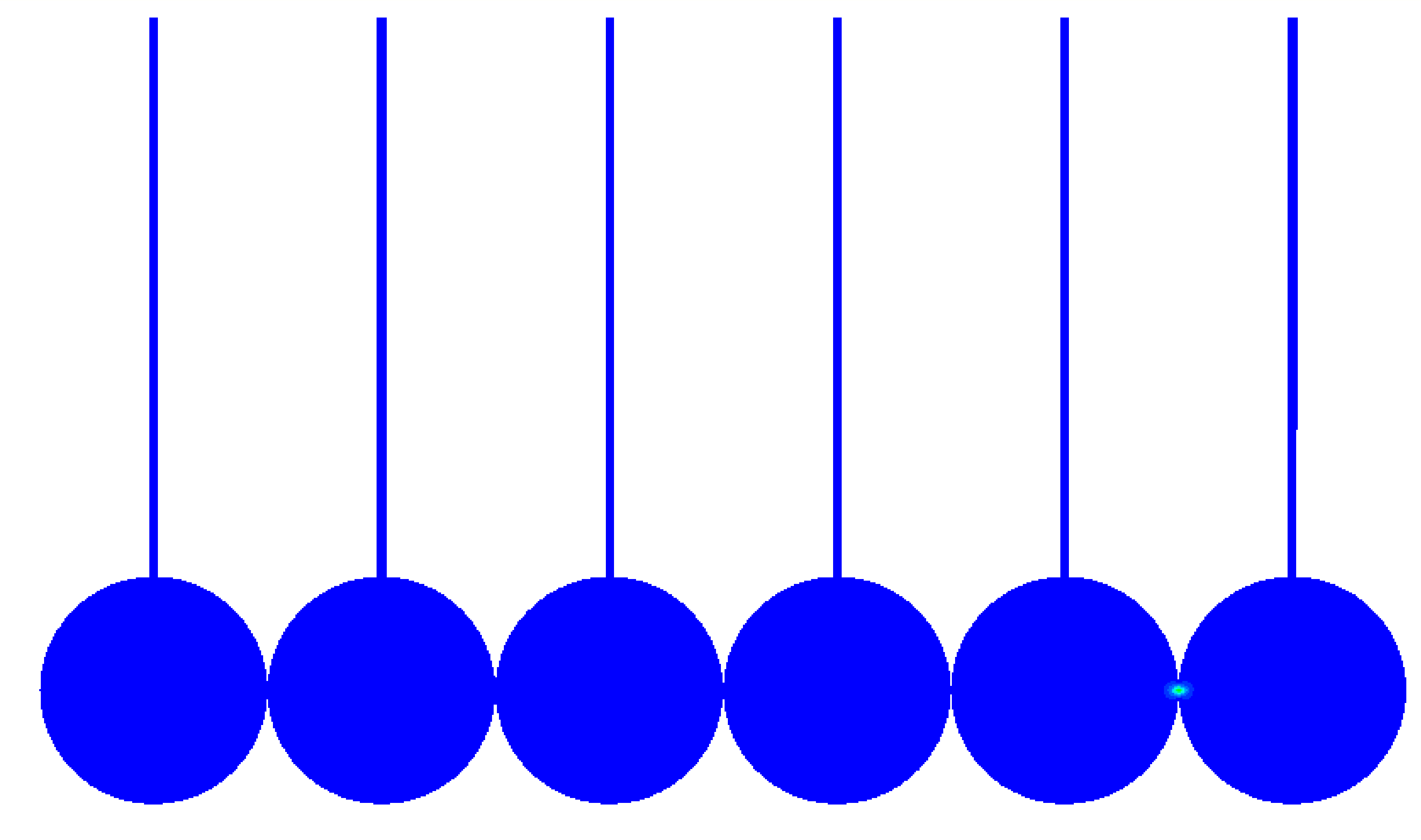




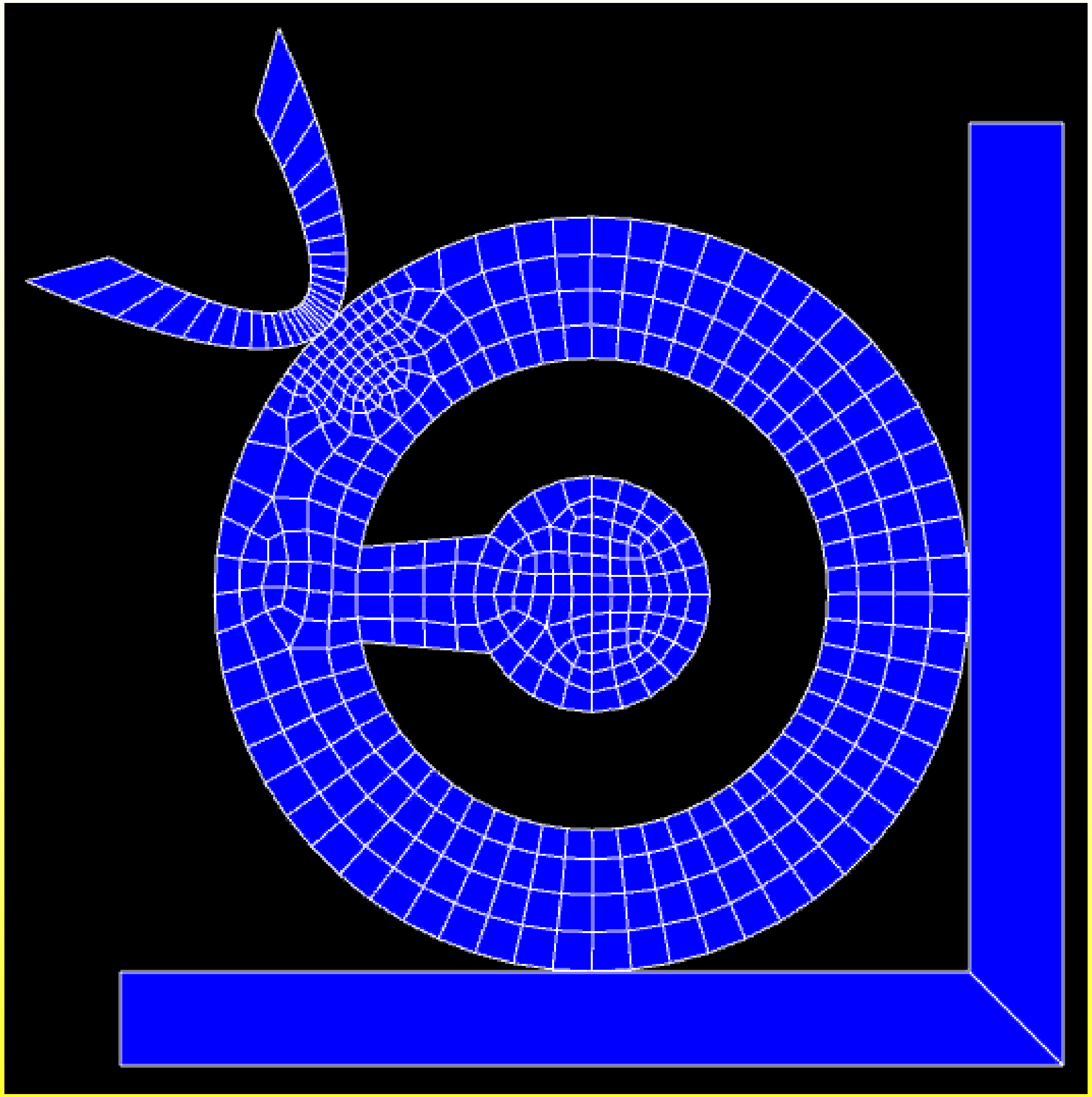
No !

◆ Is the dissipated energy monotone to the friction coefficient ?

Shock simulation



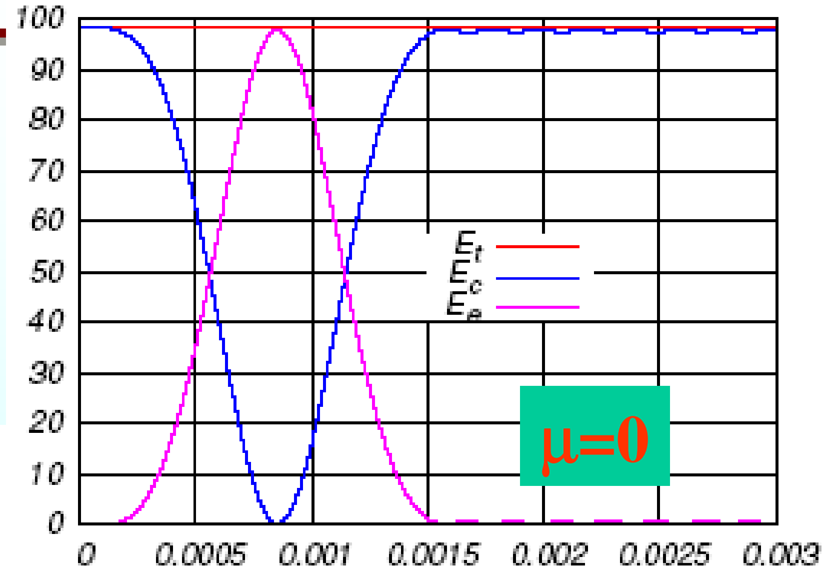
Quasi-static contact

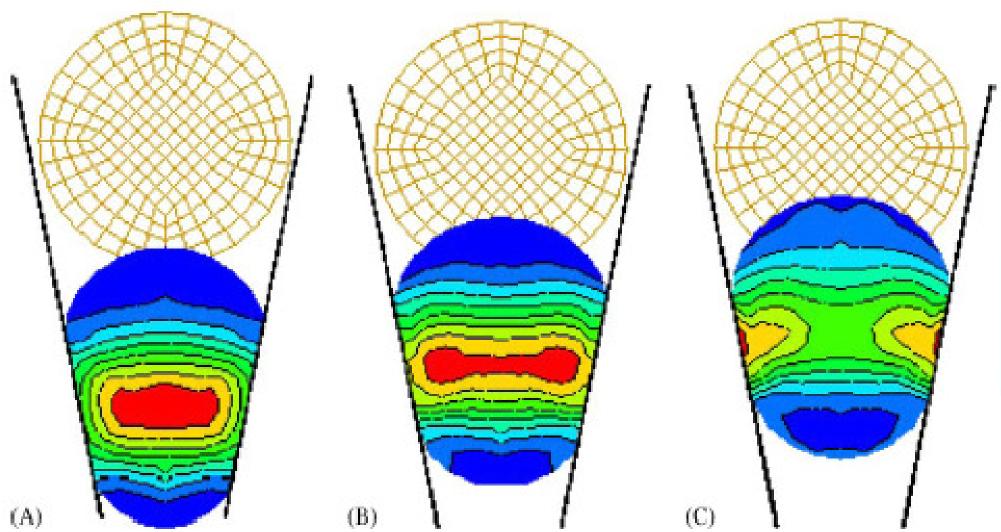


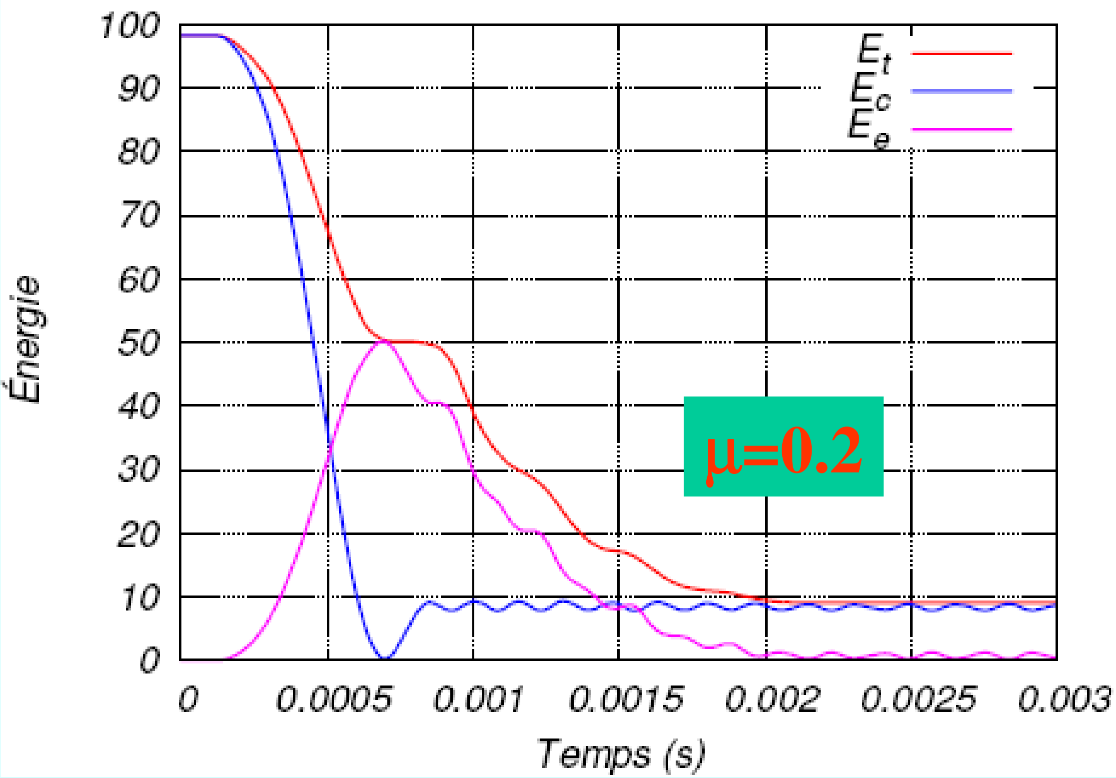
Rubber materials: Mooney-Rivlin model

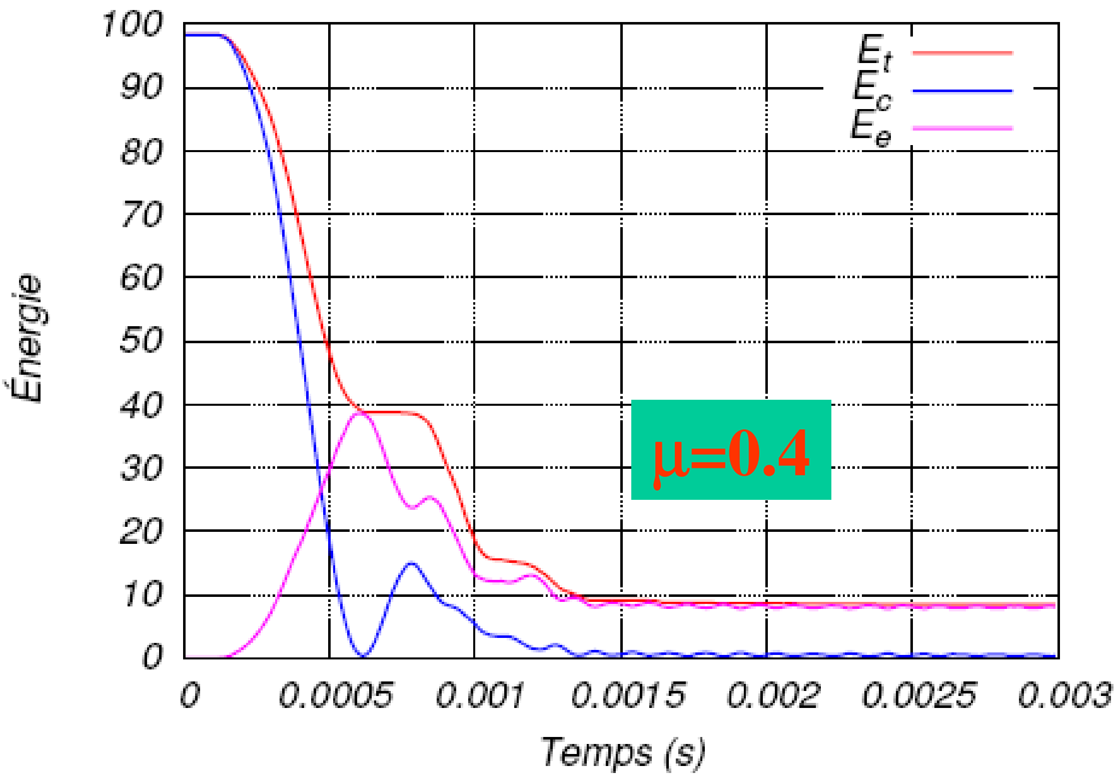
Blocking due to friction (Ogden materials)

Demo with
FER/View







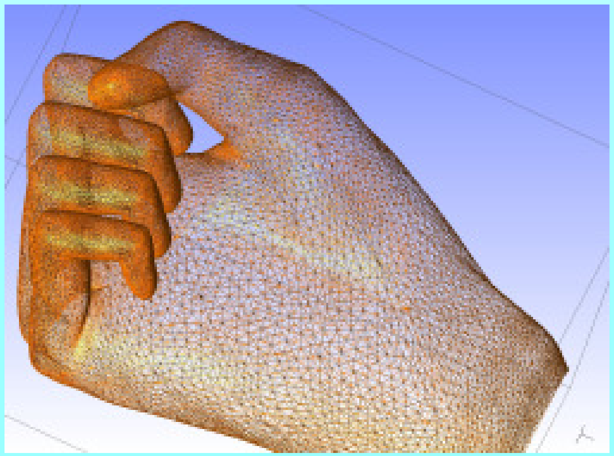


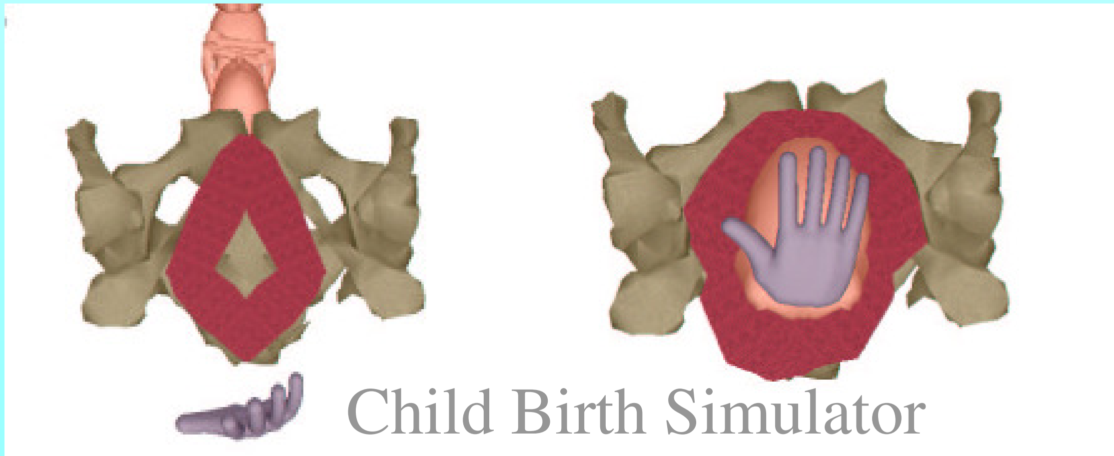
Concluding remarks

- The implicit algorithm allows for very good solution of impact problems without user-defined parameters, it can be used to solve low and high velocity impact, but it is necessary to factorize the global matrix and to perform iterations.
- The explicit algorithm needs much less memory and computational cost (no global matrix, no iteration), but the accuracy of solution depends highly on some parameters: time step, gap, hourglass, ...
- The explicit algorithm is designed rather for the modeling of high velocity impact.
- Suggestion:
  - to integrate explicitly the equation of motion
  - to solve implicitly frictional contact problems.

Some other ongoing research work

- **Variable friction: $\mu(p, v_t, T)$**
- **Modeling of wear and friction induced stability**
- **Contact-adhesion (interface, cell mechanics)**
- **Modeling in bio-mechanics:**
  - **biological soft tissues (anisotropic law, large strain)**
  - **pelvic organs interaction**
- **Virtual reality and physics-based real time simulation**





For more detail …

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